

# CANCER GENOMICS

## Lecture 2:

# Probabilistic Methods for Mutation Detection

**GENOME 541**

Spring 2020



**FRED HUTCH**  
CURES START HERE®

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[GavinHaLab.org](http://GavinHaLab.org)

# Outline

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## 1. Primer on statistical modeling (cont'd)

- Mixture models, inference and parameter estimation using the EM algorithm

## 2. Detecting Mutations in Cancer Genomes

- Visualizing somatic vs germline SNVs
- Sequencing read count data

## 3. Mixture Models for SNV Detection

- SNV genotyping strategy
- SNVMix probabilistic model and EM inference
- Predicting somatic SNVs in cancer

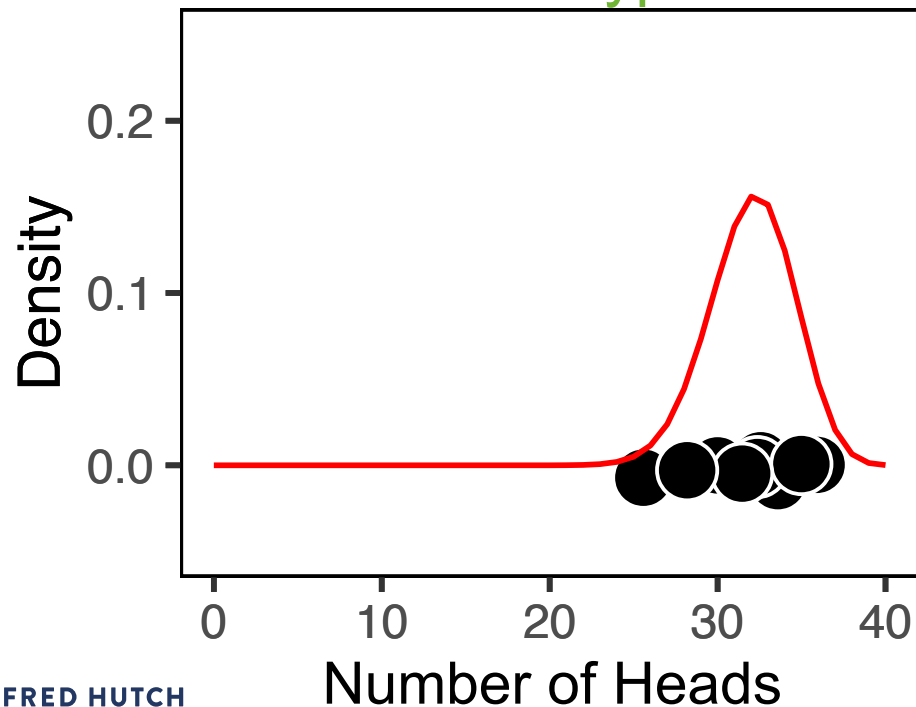
# 1. Primer on statistical modeling (cont'd)

- Probability
  - Unsupervised learning, probability rules & Bayes' theorem
  - Binomial distribution, Bayesian statistics
  - Beta-binomial model example
- **Mixture models, EM inference & parameter learning**
- References:
  - Murphy, K. (2012). Machine Learning: A Probabilistic Perspective. MIT Press. ISBN: 9780262018029
  - Bishop, C. M. (2006). Pattern Recognition and Machine Learning (Information Science and Statistics). Springer. ISBN: 0387310738

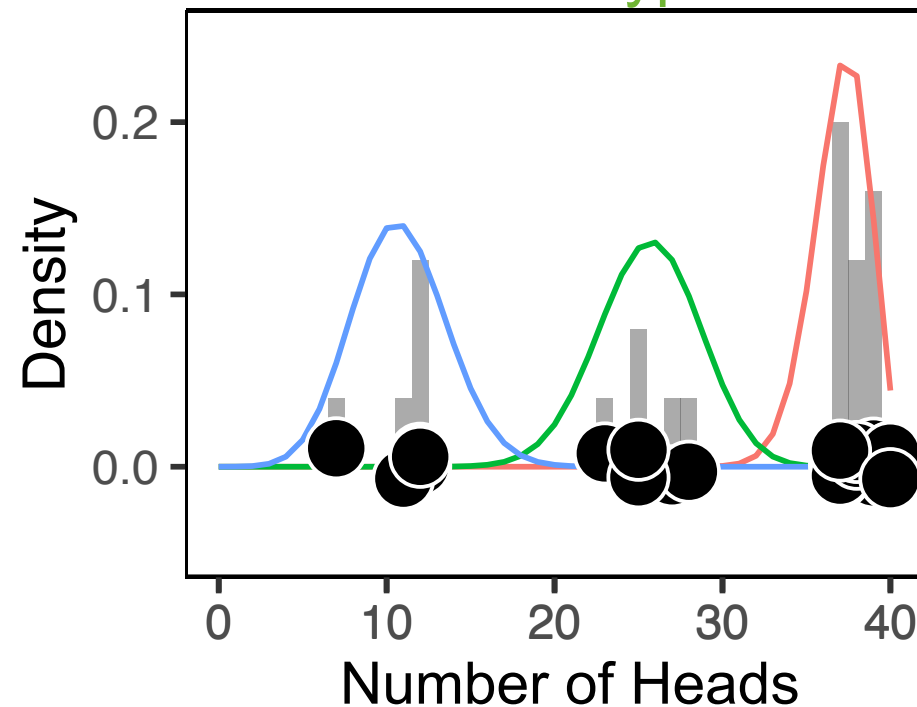
# Mixture Model: Referee example with multiple coins

- Recall: There are  $T$  different referees who tossed the *same* coin  $N = \{1, \dots, N_T\}$  times and came up with counts of heads  $\mathbf{x} = \{1, \dots, x_T\}$ .
- Now suppose there are **3 types of coins**: (1) probably fair, (2) unfairly favors heads, (3) unfairly favors tails denoted as  $\{fair, heads, tails\}$ .
- Each referee **draws one coin** (with replacement) from a hat containing these coin types mixed together.

1 coin type



3 coin types



# Mixture Model: Referee example with multiple coins

- Recall: There are  $T$  different referees who tossed the *same* coin  $N = \{1, \dots, N_T\}$  times and came up with counts of heads  $\mathbf{x} = \{1, \dots, x_T\}$ .
- Now suppose there are **3 types of coins**: (1) probably fair, (2) unfairly favors heads, (3) unfairly favors tails denoted as  $\{fair, heads, tails\}$ .
- Each referee **draws one coin from a hat** that contains a bunch of these coin types mixed together.
  - We don't know the proportion of each coin type in the hat.
  - We don't know which coin each referee drew from the hat.
  - We don't know the fairness (probability of heads) for each type of coin.

Referee	# of tosses ( $N$ )	# of heads ( $x$ )	Prop. of heads	Type of coin used?
Referee 1	40	25	0.63	?
Referee 2	42	35	0.83	?
Referee 3	39	27	0.69	?
Referee 4	$x_T$	$N_T$	$x_T/N_T$	?

Coin Type	Proportion in hat	Prob. of heads
“Fair”	?	?
“Heads”	?	?
“Tails”	?	?

# Mixture Model: Latent state model

## 1. What is the proportion of each coin type in the hat?

Find the probability for drawing a coin type.

- $\pi_k$  is the probability of drawing coin type  $k \in \{fair, heads, tails\}$
- $\boldsymbol{\pi} = (\pi_{fair}, \pi_{heads}, \pi_{tails})$  are the *mixture weights* where  $\sum_{k=1}^K \pi_k = 1$

Coin Type	Proportion in hat	Prob. of heads
“Fair”	$\pi_{fair}$	?
“Heads”	$\pi_{heads}$	?
“Tails”	$\pi_{tails}$	?

## 2. Which coin did each referee draw?

Define the latent variables.

- Let  $Z_i = k$  be the type of coin that referee  $i$  draws
- $Z_i$  is called a **latent variable** and follows a *Categorical* distribution with parameter  $\boldsymbol{\pi}$

$$p(Z_i = k | \boldsymbol{\pi}_{1:K}) = \text{Cat}(Z_i = k | \boldsymbol{\pi}_{1:K})$$

$$= \begin{cases} \pi_{fair} & \text{if } k = fair \\ \pi_{heads} & \text{if } k = heads \\ \pi_{tails} & \text{if } k = tails \end{cases}$$

- The proportions  $\boldsymbol{\pi}_{1:K}$  of the coin types follows a Dirichlet distribution (conjugate prior)

$$p(\boldsymbol{\pi}_{1:K} | \boldsymbol{\delta}_{1:K}) = \text{Dirichlet}(\boldsymbol{\pi}_{1:K} | \boldsymbol{\delta}_{1:K})$$

Referee	Type of coin used?
Referee 1	$Z_1$
Referee 2	$Z_2$
Referee 3	$Z_3$
Referee T	$Z_T$



# Mixture Model: Likelihood as a mixture of binomials

3. What is the fairness (prob. of heads) for each type of coin?

Find the probability of heads for each coin type.

- Recall: for a single coin,  $p(x_i | N_i, \mu) = \text{Bin}(x_i | N_i, \mu)$
- Define the likelihood for a **3-component mixture of binomials** with 3 parameters,  $\mu_{fair}, \mu_{heads}, \mu_{tails}$ , one for each type of coin

$$p(x_i | Z_i = k, N_i, \mu_{1:K}) = \text{Bin}(x_i | N_i, \mu_k)$$

$$p(x_i | N_i, \mu_{1:K}, \pi_{1:K}) = \sum_{k=1}^K \pi_k \text{Bin}(x_i | N_i, \mu_k)$$

- Beta prior distribution  $p(\mu_k | \alpha_k, \beta_k) = \text{Beta}(\mu_k | \alpha_k, \beta_k)$

Log Likelihood Function of the Model

$$L(x_{1:T}, N_{1:T} | \mu_{1:K}, \pi_{1:K}) = \prod_{i=1}^T \sum_{k=1}^K \pi_k \text{Bin}(x_i | N_i, \mu_k)$$

$$\ell = \sum_{i=1}^T \log \left( \sum_{k=1}^K \pi_k \text{Bin}(x_i | N_i, \mu_k) \right)$$

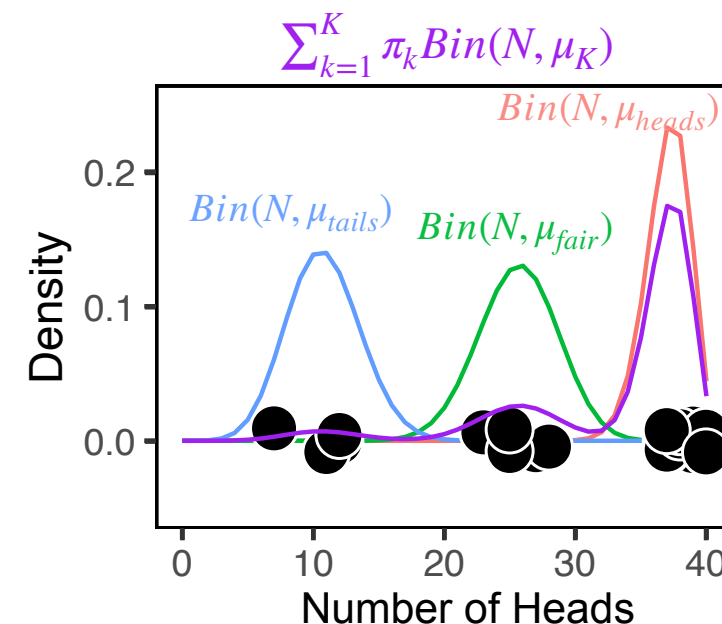
Coin Type	Proportion in hat	Prob. of heads
“Fair”	$\pi_{fair}$	$\mu_{fair}$
“Heads”	$\pi_{heads}$	$\mu_{heads}$
“Tails”	$\pi_{tails}$	$\mu_{tails}$

Observed likelihood

Mixture model

Likelihood function

Log likelihood



# Mixture Model: Inference & parameter estimation using EM (1)

## Expectation-Maximization: Inference and parameter training

Initialize parameters:  $\pi_{1:K}$  and  $\mu_{1:K}$

### E-Step: compute “responsibilities” (inference)

1. Which coin did each referee draw?

(Posterior of the latent states  $\gamma(Z_{1:T})$ )

- Soft-clustering: Referee  $i$  has a probability for using each of the coins.
- responsibilities: “coin that is responsible for generating observation  $x_i$ ”

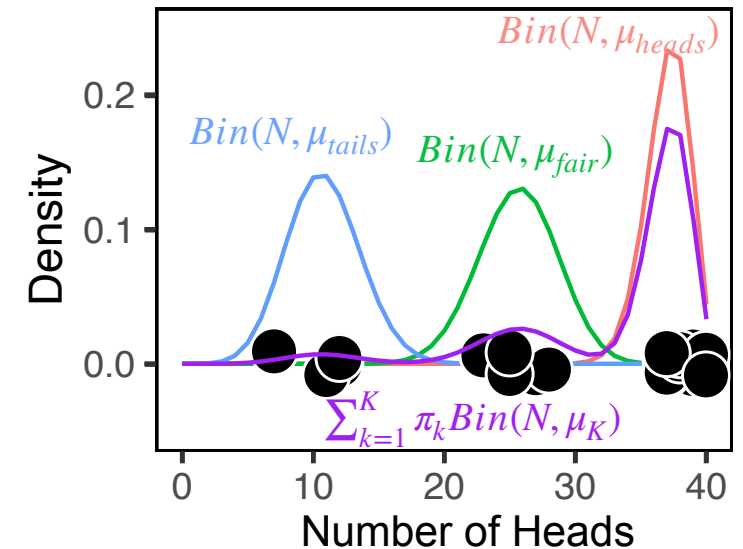
### M-Step: Update parameters (learning)

2. What is the proportion of each coin type in the hat?  $\pi_{1:K}$

3. What is the fairness (prob. of heads) for each coin type?  $\mu_{1:K}$

Iterate between E-Step and M-Step, check when log-likelihood  $\ell$  stops increasing.

Responsibilities			
Referee	Fair Coin	Heads Coin	Tails Type Coin
1	$\gamma(Z_1 = F)$	$\gamma(Z_1 = H)$	$\gamma(Z_1 = T)$
2	$\gamma(Z_2 = F)$	$\gamma(Z_2 = H)$	$\gamma(Z_2 = T)$
3	$\gamma(Z_3 = F)$	$\gamma(Z_3 = H)$	$\gamma(Z_3 = T)$
T	$\gamma(Z_T = F)$	$\gamma(Z_T = H)$	$\gamma(Z_T = T)$





# Mixture Model: Inference & parameter estimation using EM (2)

## E-Step: compute responsibilities (inference)

1. What is the probability for a referee to draw each coin type? (Posterior of the latent states  $Z_{1:T}$ )

- Find the responsibilities given the current parameters

$$\begin{aligned} p(Z_i = k | x_i, N_i, \pi_{1:K}, \mu_{1:K}) &= \frac{p(x_i | Z_i = k)p(Z_i = k)}{p(x_i)} \\ &= \frac{\pi_k \text{Bin}(x_i | N_i, \mu_k)}{\sum_{k'=1}^K \pi_{k'} \text{Bin}(x_i | N_i, \mu_{k'})} \\ &= \gamma(Z_i = k) \end{aligned}$$

**Bayes' Rule**  
**Posterior distribution**  
**of the latent variables**

**Responsibilities**  
Matrix  $T \times K$

- Responsibilities = “coin that is responsible for generating observation  $x_i$ ”
- Soft-clustering: Referee  $i$  has a probability for using each of the coins.
- $\gamma(Z_{1:T})$  is a matrix of probabilities with dimensions  $T \times K$

# Mixture Model: Inference & parameter estimation using EM (3)

## M-Step: Update parameters (learning)

2. What is the proportion of each coin type in the hat?

$$\hat{\pi}_k = \frac{\sum_{i=1}^T \gamma(Z_i = k) + \delta(k) - 1}{\sum_{j=1}^K \sum_{i=1}^T \{\gamma(Z_i = j) + \delta(j) - 1\}}$$

MAP for  $\pi$

3. What is the fairness (prob. of heads) for each coin type?

$$\hat{\mu}_k = \frac{\sum_{i=1}^T \gamma(Z_i = k)x_i + \alpha_k - 1}{\sum_{i=1}^T \gamma(Z_i = k)N_i + \alpha_k + \beta_k - 2}$$

MAP for  $\mu$

Evaluate the log likelihood and log posterior: use updated parameters

**Log posterior**  $\log \mathbb{P} = \underbrace{\sum_{i=1}^T \log \left( \sum_{k=1}^K \hat{\pi}_k \text{Bin}(x_i | N_i, \hat{\mu}_k) \right)}_{\text{Log likelihood}} + \underbrace{\log \text{Dir}(\hat{\pi} | \delta) + \sum_{k=1}^K \log \text{Beta}(\hat{\mu}_k | \alpha_k, \beta_k)}_{\text{Log priors}}$

Iterate between E-Step and M-Step:

- Stop EM when new  $\log \mathbb{P}$  changes less than  $\epsilon$  compared to previous EM iteration.

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**Algorithm 1** Binomial Mixture Model Inference and Learning using EM

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```
1: Inputs:  
   Data:  $x_{1:T}, N_{1:T}$   
   Initial parameters:  $\pi_{1:K}^{(0)}, \mu_{1:K}^{(0)}$   
   Hyperparameters:  $\delta_{1:K}, \alpha_{1:K}, \beta_{1:K}$   
2: Initialize:  
    $\pi_{1:K} \leftarrow \pi_{1:K}^{(0)}, \mu_{1:K} \leftarrow \mu_{1:K}^{(0)}$   
3:    $\log P \leftarrow -Inf$   
4: Compute the observed likelihood using initial parameters:  
5:    $lik \leftarrow \text{compute.binom.lik}()$   
6: while converged = false do  
7:   E-Step: Compute responsibilities:  
8:      $\gamma(Z_{1:T}) \leftarrow \text{compute.responsibilities}()$   
9:   M-Step: Update parameters:  
10:     $\hat{\pi}_{1:K} \leftarrow \text{update.pi}()$   
11:     $\hat{\mu}_{1:K} \leftarrow \text{update.mu}()$   
12:   Assign updated parameters:  
13:     $\pi_{1:K} \leftarrow \hat{\pi}_{1:K}, \mu_{1:K} \leftarrow \hat{\mu}_{1:K}$   
14:   Re-compute the observed likelihood using updated parameters:  
15:     $\text{obs.lik} \leftarrow \text{compute.binom.lik}()$   
16:   Compute the log-likelihood:  
17:     $\text{loglik} \leftarrow \text{compute.loglik}()$   
18:   Compute log Posterior:  
19:     $\text{logP[curr.iter]} \leftarrow \text{compute.log.posterior}()$   
20:   if (  $\text{logP[curr.iter]} - \text{logP[prev.iter]} < \epsilon$  ) then  
21:     converged = true  
22:   end if  
23:    $\text{logP[prev.iter]} \leftarrow \text{logP[curr.iter]}$   
24: end while  
25: return Responsibilities  $\gamma(Z_{1:T})$ , Converged parameters  $\hat{\pi}_{1:K}, \hat{\mu}_{1:K}$ 
```

# Mixture Model: Inference & parameter estimation using EM (extra slide 1)

## Incomplete data log likelihood

$$L(x_{1:T}, N_{1:T} | \mu_{1:K}, \pi_{1:K}) = \prod_{i=1}^T \sum_{k=1}^K \pi_k \text{Bin}(x_i | N_i, \mu_k)$$

- The incomplete data log likelihood (plus the priors) is used to monitor EM convergence

## Expected complete data log likelihood

**Complete data likelihood**

$$L(\mu_{1:K}, \pi_{1:K} | x_{1:T}, Z_{1:T}, N_{1:T}) = \prod_{i=1}^T \prod_{k=1}^K \pi_k \text{Bin}(x_i | N_i, \mu_k)^{\mathbb{1}(Z_i=k)}$$

**Complete data log likelihood**

$$\ell(\mu_{1:K}, \pi_{1:K} | x_{1:T}, Z_{1:T}, N_{1:T}) = \sum_{i=1}^T \sum_{k=1}^K \mathbb{1}(Z_i = k) \{ \log \pi_k + \log \text{Bin}(x_i | N_i, \mu_k) \}$$

**Expected complete data log likelihood**

$$\begin{aligned} Q = \mathbb{E} [\ell(\mu_{1:K}, \pi_{1:K} | x_{1:T}, Z_{1:T}, N_{1:T})] &= \sum_{i=1}^T \sum_{k=1}^K \mathbb{E} [\mathbb{1}(Z_i = k)] \{ \log \pi_k + \log \text{Bin}(x_i | N_i, \mu_k) \} \\ &= \sum_{i=1}^T \sum_{k=1}^K \gamma(Z_i = k) \{ \log \pi_k + \log \text{Bin}(x_i | N_i, \mu_k) \} \end{aligned}$$

- The expected complete data log likelihood is in the M-Step for updating parameters.

# Mixture Model: Inference & parameter estimation using EM (extra slide 2)

**M-Step:** Update the parameters given the responsibilities

$$\mathbb{P}(\pi_{1:K}, \mu_{1:K}) = Dir(\boldsymbol{\pi} | \boldsymbol{\delta}) \prod_{k=1}^K Beta(\mu_k | \alpha, \beta) \quad \text{Priors}$$

$$\mathcal{O} = Q + \log \mathbb{P}(\pi_{1:K}, \mu_{1:K}) \quad \text{Complete data log likelihood} \\ + \text{log priors}$$

- The object function  $\mathcal{O}$  is used to obtain the update equations for  $\pi_{1:K}$  and  $\mu_{1:K}$

$$\frac{\partial \mathcal{O}}{\partial \mu_k} = 0, \text{ find } \hat{\mu}_k \text{ and } \frac{\partial \mathcal{O}}{\partial \pi_k} = 0, \text{ find } \hat{\pi}_k$$

**EM Convergence:** after each iteration, monitor the log posterior

$$\ell = \sum_{i=1}^T \log \left( \sum_{k=1}^K \pi_k Bin(x_i | \mu_k, N_i) \right) \quad \text{Incomplete Data} \\ \text{Log likelihood}$$

$$\log \mathbb{P}(\pi_{1:K}, \mu_{1:K} | x_{1:T}) = \ell + \log \mathbb{P}(\pi_{1:K}, \mu_{1:K}) \quad \text{Log posterior}$$

- If the log posterior,  $\log \mathbb{P}(\pi_{1:K}, \mu_{1:K} | x_{1:T})$ , stops increasing by  $\epsilon$ , then EM is converged.
- If not using a Bayesian framework, then use the log likelihood,  $\ell$ , to monitor convergence.

# Mixture Models: Online Tutorial and Resource

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**fiveMinuteStats** (<https://stephens999.github.io/fiveMinuteStats/>)

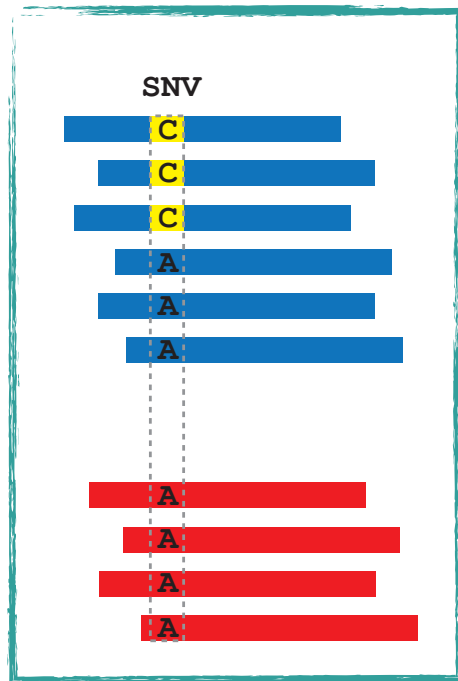
by **Dr. Matthew Stephens**, Professor in Statistics & Human Genetics at University of Chicago

1. Introduction to mixture models with probabilistic derivations and R code
  - Examples with Bernoulli and Gaussian models
  - [https://stephens999.github.io/fiveMinuteStats/intro\\_to\\_mixture\\_models.html](https://stephens999.github.io/fiveMinuteStats/intro_to_mixture_models.html)
2. Introduction to EM with Gaussian Mixture Model example and R code
  - [https://stephens999.github.io/fiveMinuteStats/intro\\_to\\_em.html](https://stephens999.github.io/fiveMinuteStats/intro_to_em.html)

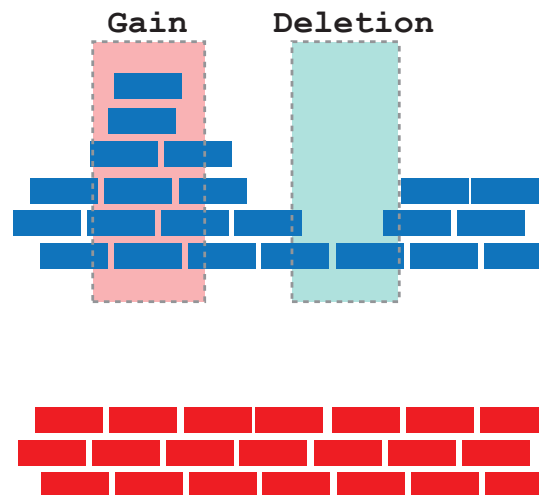


## 2. Detecting Mutations in Cancer Genomes

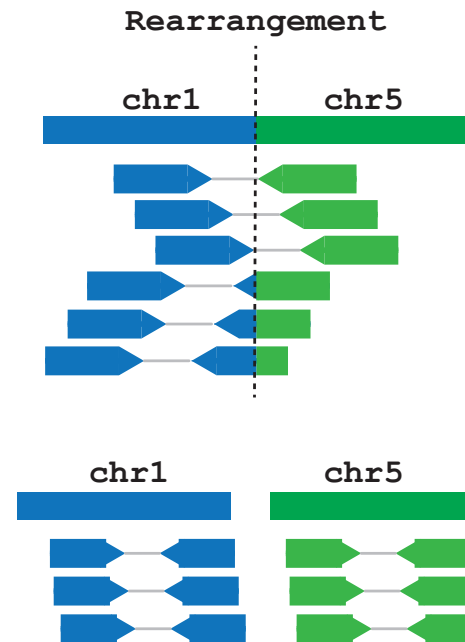
### Mutations (SNV, INDEL)



### Copy Number Alterations



### Structural Variants



# Visual inspection using IGV: Germline SNVs

## Integrative Genomics Viewer (<https://software.broadinstitute.org/software/igv>)

- ~1.5 to 2 million **SNPs** per individual
- Identify SNPs from normal peripheral blood mononuclear cells (PBMC)



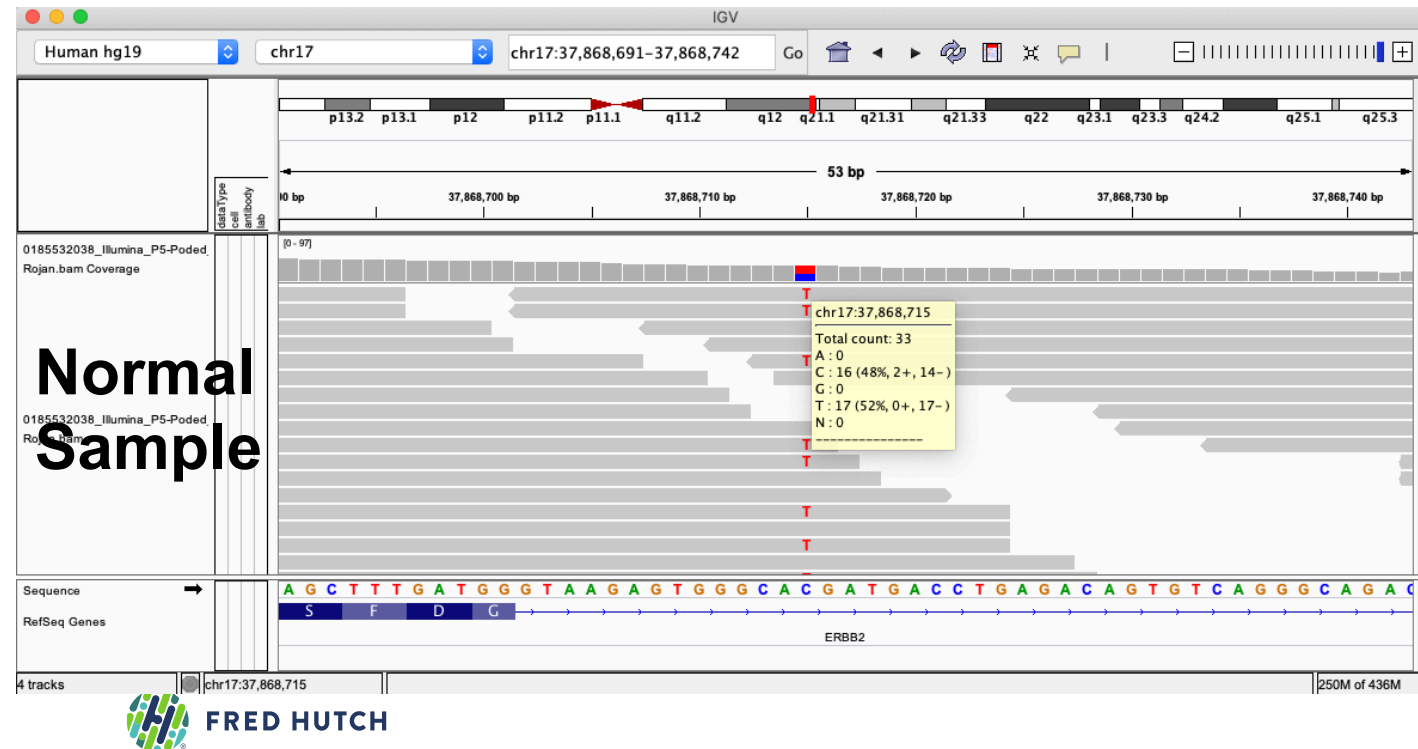
Heterozygous SNP with 17 reads containing the variant and having depth 33 reads

17/33 (48%) variant allele fraction (VAF)

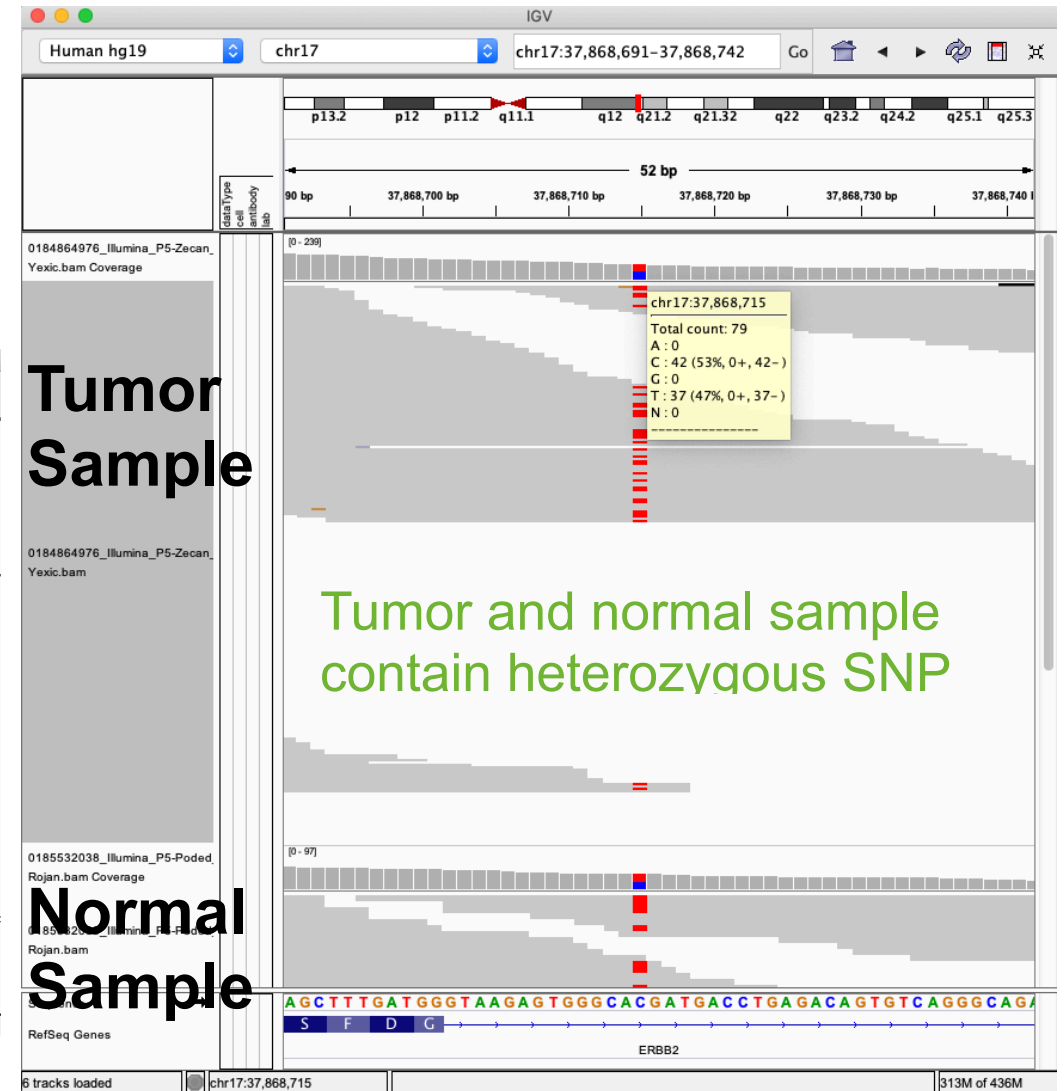
# Visual inspection using IGV: Germline SNVs

## Integrative Genomics Viewer (<https://software.broadinstitute.org/software/igv>)

- ~1.5 to 2 million **SNPs** per individual
- Identify SNPs from normal peripheral blood mononuclear cells (PBMC)



**Normal Sample**



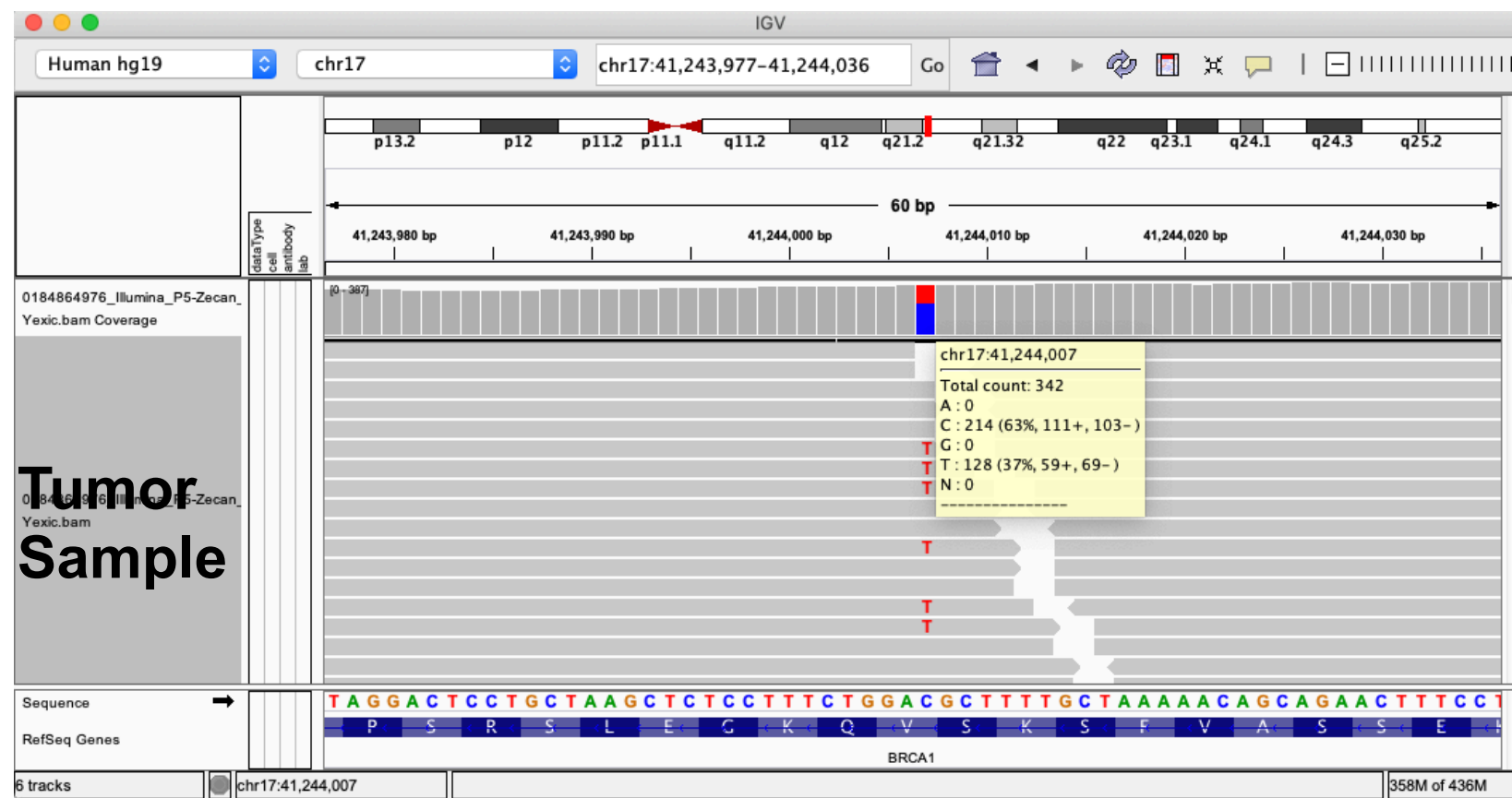
**Tumor Sample**

Tumor and normal sample contain heterozygous SNP

**Normal Sample**

# Visual inspection using IGV: Somatic SNVs

- Somatic **SNV** requires comparing case (tumor) with control (PBMC)
- On the order of 10 to  $10^4$  number of mutations

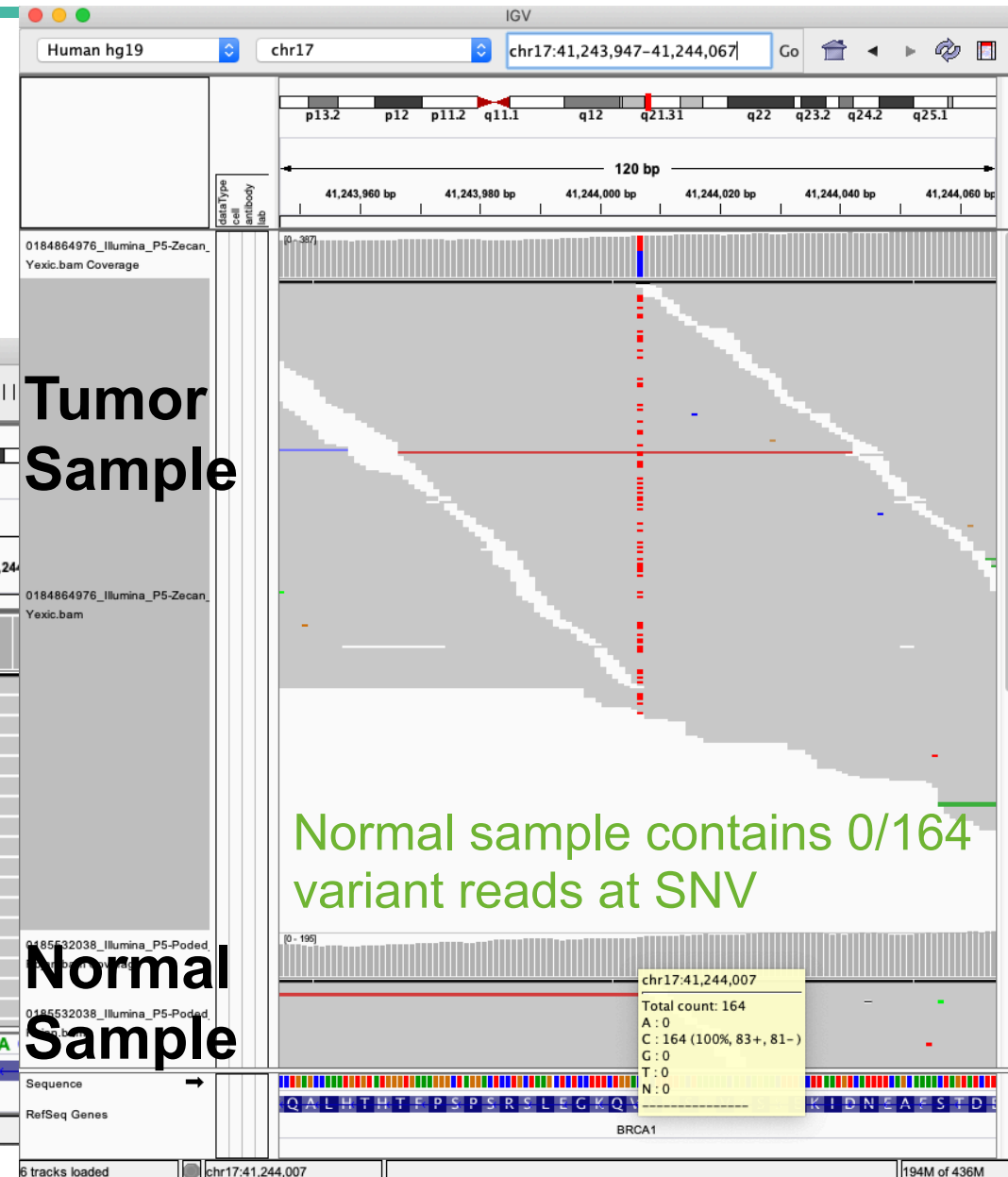
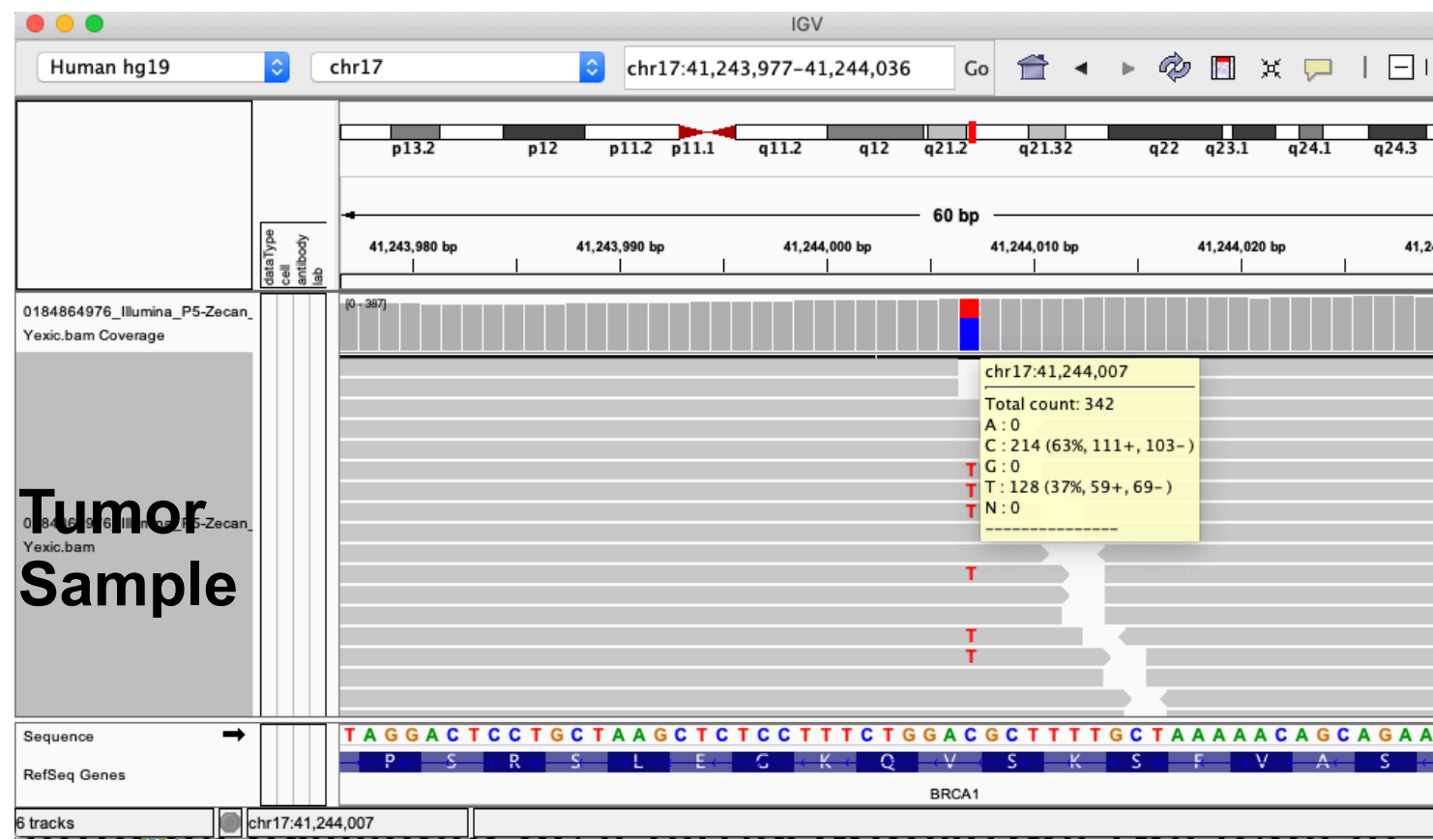


Potential SNV with  
128/342 (37%) VAF

p.V1181I

# Visual inspection using IGV: Somatic SNVs

- Somatic **SNV** requires comparing case (tumor) with control (PBMC)
- On the order of  $10$  to  $10^4$  number of mutations



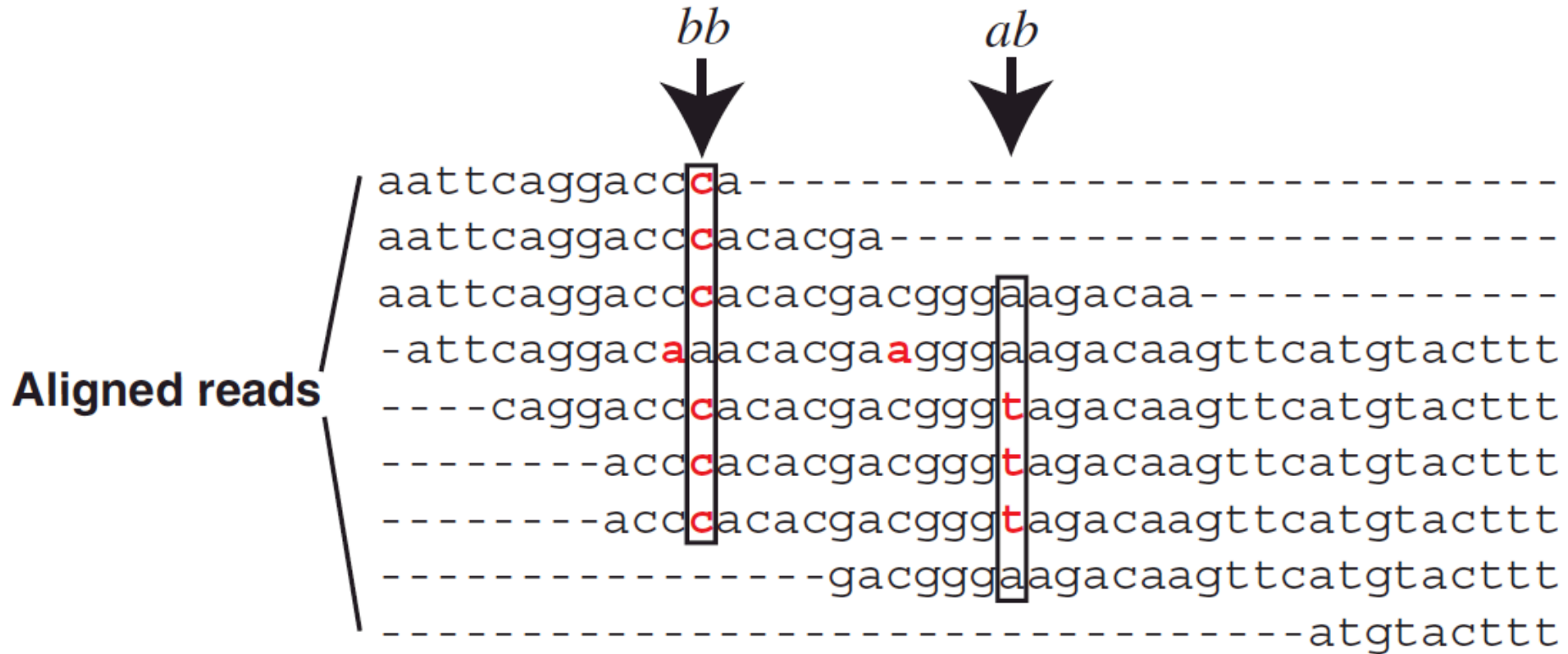
Tumor Sample

Normal Sample

Normal sample contains 0/164 variant reads at SNV

Tumor Sample

# Single Nucleotide Variant (SNV) Calling: Single Sample



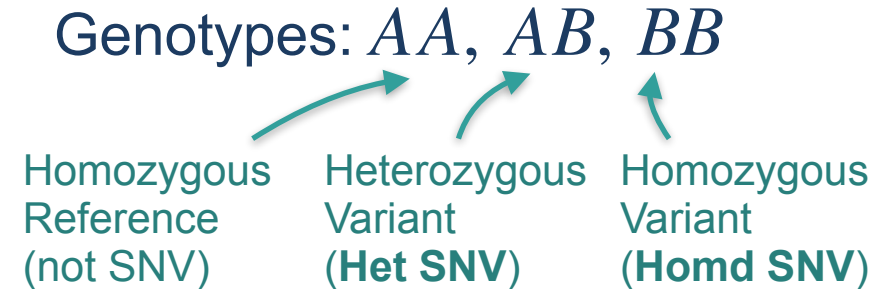
Reference seq **aattcaggaccaacacgacggggaagacaagttcatgtacttt**

Allelic counts	<b>a</b>	34445555776	<b>1</b>	7666775666	<b>3</b>	666666555566666666	Reference Counts
	<b>b</b>	0000000000	<b>1</b>	60000001000	<b>3</b>	00000000000000000000	Non-reference Counts



# SNV Variant Allele Fraction and Genotypes

## Variant Allele Fraction (VAF) Analysis



Genotype	AA	AB	BB
Allelic Fraction	~1.0	~0.5	~0

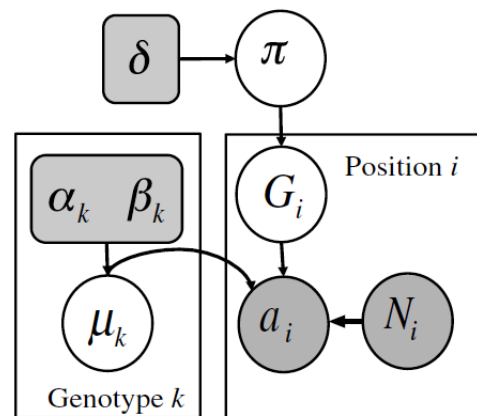
- **Allelic Fraction** is defined as the fraction of reference reads,  $\frac{A}{N}$ , where depth  $N = A + B$
- Values in the table are the *expected* proportions of *reference reads* for each genotype
- Why might the observed allelic fractions be different than the expected values?

### 3. Mixture Model for SNV Detection

- SNVMix probabilistic model and EM inference
- Predicting somatic SNVs in cancer

References:

- Goya et al. **SNVMix**: predicting single nucleotide variants from next-generation sequencing of tumors. *Bioinformatics* **26**:730-36 (2010)
- Roth et al. **JointSNVMix**: a probabilistic model for accurate detection of somatic mutations in normal/tumour paired next-generation sequencing data. *Bioinformatics* **28**:907-13 (2012)



SNVMix1 model

# Mapping the Referee Example to Mutation Calling

## Referee Coin Toss Example

### Data

Referees  $1, \dots, T$

For each Referee  $i$

- Coin Tosses:  $N_i$
- Count of heads:  $x_i$
- Count of tails:  $N_i - x_i$

### Parameters

Probability to draw coins:  $\pi_{fair}, \pi_{heads}, \pi_{tails}$

Probability of heads for 3 types of coins

$$\mu_{fair}, \mu_{heads}, \mu_{tails}$$

### Responsibilities

Probability that Referee  $i$  used coin  $k$ :  $\gamma(Z_i = k)$

## Mutation Calling from Sequencing Data

### Data

Genomic loci  $1, \dots, T$

For each locus  $i$

- Depth (total reads):  $N_i$
- Count of reference base:  $x_i$
- Count of variant base:  $N_i - x_i$

### Parameters

Probability of genotypes:  $\pi_{AA}, \pi_{AB}, \pi_{BB}$

Probability of reference base for 3 genotypes:

$$\mu_{AA}, \mu_{AB}, \mu_{BB}$$

### Responsibilities

Probability that locus  $i$  has genotype  $k$ :  $\gamma(Z_i = k)$

# SNVMix: Probabilistic Model

## Sequence Data

There are  $T$  different genomic loci with read depths  $N = \{1, \dots, N_T\}$  and reference base counts  $\mathbf{x} = \{1, \dots, x_T\}$ .  
There are  $K = 3$  different possible genotypes  $AA, AB, BB$

## Mixture Model Setup

1. The **probabilities for the genotypes** are  $\pi_{AA}, \pi_{AB}, \pi_{BB}$

2. Thus, a specific genotype  $k \in AA, AB, BB$  can be assigned to the **latent state**  $Z_i$  at locus  $i$  with these probabilities

$$p(Z_i = k | \pi_{1:K}) = \begin{cases} \pi_{AA} & \text{if } k = AA \\ \pi_{AB} & \text{if } k = AB \\ \pi_{BB} & \text{if } k = BB \end{cases}$$

3. The **probability of observing a reference base** for the genotypes are  $\mu_{aa}, \mu_{ab}, \mu_{bb}$

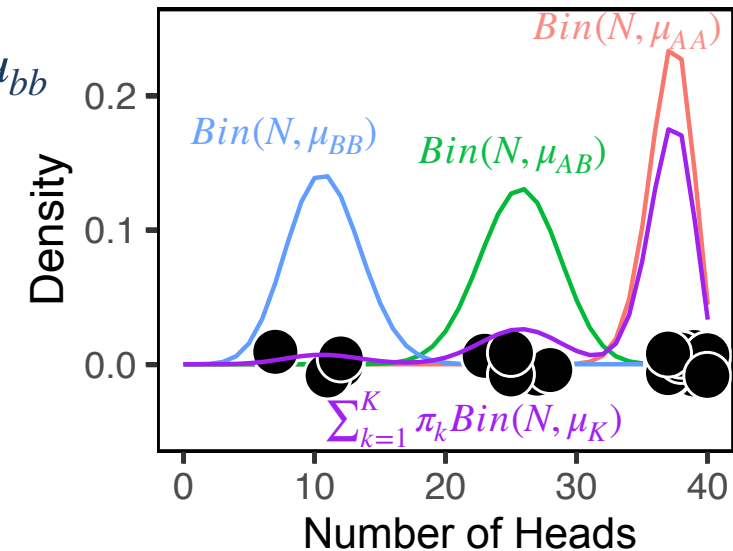
4. The likelihood is a **3-component mixture of binomials**

$$p(x_i | N_i, \mu_{1:K}, \pi_{1:K}) = \sum_{k=1}^K \pi_k \text{Bin}(x_i | N_i, \mu_k)$$

5. The **priors** for genotype  $k \in \{aa, ab, bb\}$  in the model are

$$p(\pi_{1:K} | \delta_{1:K}) = \text{Dirichlet}(\pi_{1:K} | \delta_{1:K})$$

$$p(\mu_k | \alpha_k, \beta_k) = \text{Beta}(\mu_k | \alpha_k, \beta_k)$$



# SNVMix: Inference & parameter estimation using EM (revisited)

## E-Step: compute responsibilities

1. What is the probability of locus  $i$  having genotype  $k$ ?

$$\gamma(Z_i = k) = \frac{\pi_k \text{Bin}(x_i | N_i, \mu_k)}{\sum_{j=1}^K \pi_j \text{Bin}(x_i | N_i, \mu_j)}$$

**Responsibilities**

Matrix  $T \times K$

## M-Step: update parameters

2. What is the probability of genotype  $k$ ?

$$\hat{\pi}_k = \frac{\sum_{i=1}^T \gamma(Z_i = k) + \delta(k) - 1}{\sum_{j=1}^K \left\{ \sum_{i=1}^T \gamma(Z_i = j) + \delta(j) - 1 \right\}}$$

**MAP for  $\pi$**

3. What is the probability of observing a reference base for genotype  $k$ ?

$$\hat{\mu}_k = \frac{\sum_{i=1}^T \gamma(Z_i = k) x_i + \alpha_k - 1}{\sum_{i=1}^T \gamma(Z_i = k) N_i + \alpha_k + \beta_k - 2}$$

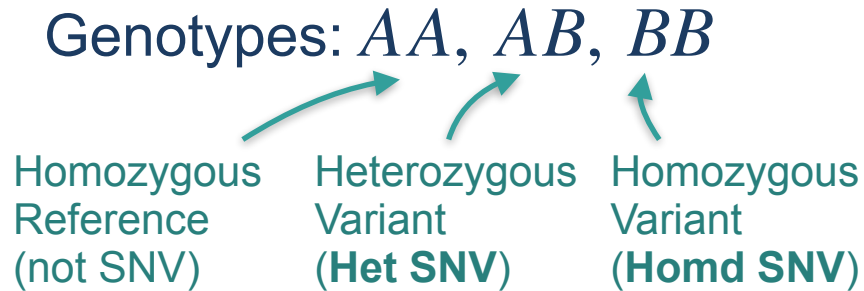
**MAP for  $\mu$**

## Evaluate the log likelihood and log posterior: use updated parameters

$$\log \mathbb{P} = \sum_{i=1}^T \log \left( \sum_{k=1}^K \hat{\pi}_k \text{Bin}(x_i | \hat{\mu}_k, N_i) \right) + \log \text{Dir}(\hat{\pi}_k | \delta_k) + \sum_{k=1}^K \log \text{Beta}(\hat{\mu}_k | \alpha_k, \beta_k) \quad \text{Log posterior}$$

**Iterate between E-Step and M-Step:** stop when  $\log \mathbb{P}$  changes less than  $\epsilon$  compared to previous EM iteration.

# SNVMix: Calling somatic SNVs from genotype inference



Responsibilities			
Locus	AA	AB	BB
1	$\gamma(Z_1 = AA)$	$\gamma(Z_1 = AB)$	$\gamma(Z_1 = BB)$
2	$\gamma(Z_2 = AA)$	$\gamma(Z_2 = AB)$	$\gamma(Z_2 = BB)$
3	$\gamma(Z_3 = AA)$	$\gamma(Z_3 = AB)$	$\gamma(Z_3 = BB)$
T	$\gamma(Z_T = AA)$	$\gamma(Z_T = AB)$	$\gamma(Z_T = BB)$

- To call a variant for each locus  $i$ , we can apply a threshold on the responsibilities  $\gamma(Z_i)$
- We can sum  $\gamma(Z_i = AB)$  and  $\gamma(Z_i = BB)$  to get the overall probability (either genotype AB or BB) that locus  $i$  is a variant containing the non-reference allele (B)
- Additional steps required for filtering and determining if variant is somatic vs germline
  - Minimum 3 variant reads ( $N_i - x_i$ ) is typically required
  - Account for mapping and base qualities of sequenced reads (i.e. SNVMix2)
  - Compare locus  $i$  in tumor sample to (1) matched normal sample, (2) germline databases



# SNV Genotyping Callers

## Variant Allele Fraction Analysis

- Single sample

Genotypes: *AA*, *AB*, *BB*

Homozygous Reference (not SNV) → *AA*  
Heterozygous Variant (Het SNV) → *AB*  
Homozygous Variant (Homd SNV) → *BB*

- Joint tumor-normal

Joint Genotypes:

$g_N \backslash g_T$	AA	AB	BB
AA	0.01	0.95	0.00
AB	0.00	0.04	0.00
BB	0.00	0.00	0.00

- Cohort level or panel: Machine Learning (supervised)

Reference Genome

ACTCCCGTCGGAAACGAATGCCACG

Normal

Allelic Counts

$a_N$  12233556666666777778773  
 $d_N$  12233566666666777778777

Tumour

Allelic Counts

$a_T$  112333445563660777788883  
 $d_T$  112333445565666777788888

█ Germline  
█ Somatic

█ (AA,AB)    █ (BB,BB)    █ (AB,AB)

Variant caller	Type of variant	Single-sample mode	Type of core algorithm
BAYSIC [48]	SNV	No	Machine learning (ensemble caller)
CaVEMan [34]	SNV	No	Joint genotype analysis
deepSNV [38]	SNV	No	Allele frequency analysis
EBCall [37]	SNV, indel	No	Allele frequency analysis
FaSD-somatic [31]	SNV	Yes	Joint genotype analysis
FreeBayes [44]	SNV, indel	Yes	Haplotype analysis
HapMuC [42]	SNV, indel	Yes	Haplotype analysis
JointSNVMix2 [30]	SNV	No	Joint genotype analysis
LocHap [43]	SNV, indel	No	Haplotype analysis
LoFreq [36]	SNV, indel	Yes	Allele frequency analysis
LoLoPicker [39]	SNV	No	Allele frequency analysis
MutationSeq [45]	SNV	No	Machine learning
MuSE [40]	SNV	No	Markov chain model
MuTect [35]	SNV	Yes	Allele frequency analysis
SAMtools [8]	SNV, indel	Yes	Joint genotype analysis
Platypus [41]	SNV, indel, SV	Yes	Haplotype analysis
qSNP [24]	SNV	No	Heuristic threshold
RADIA [26]	SNV	No	Heuristic threshold
Seurat [33]	SNV, indel, SV	No	Joint genotype analysis
Shimmer [25]	SNV, indel	No	Heuristic threshold
SNooPer [47]	SNV, indel	Yes	Machine learning
SNVsniffer [32]	SNV, indel	Yes	Joint genotype analysis
SOAPsnv [27]	SNV	No	Heuristic threshold
SomaticSeq [46]	SNV	No	Machine learning (ensemble caller)
SomaticSniper [28]	SNV	No	Joint genotype analysis
Strelka [17]	SNV, indel	No	Allele frequency analysis
TVC [97]	SNV, indel, SV	Yes	Ion Torrent specific
VarDict [18]	SNV, indel, SV	Yes	Heuristic threshold
VarScan2 [9]	SNV, indel	Yes	Heuristic threshold
Virmid [29]	SNV	No	Joint genotype analysis

# Somatic SNV Detection using Joint Inference from Tumor-Normal Pairs

## 1. Latent variable state space

- 9 genotype pairs  $(k_n, k_t)$
- $n, t \in \{AA, AB, BB\}$

## 2. Probability of the genotypes

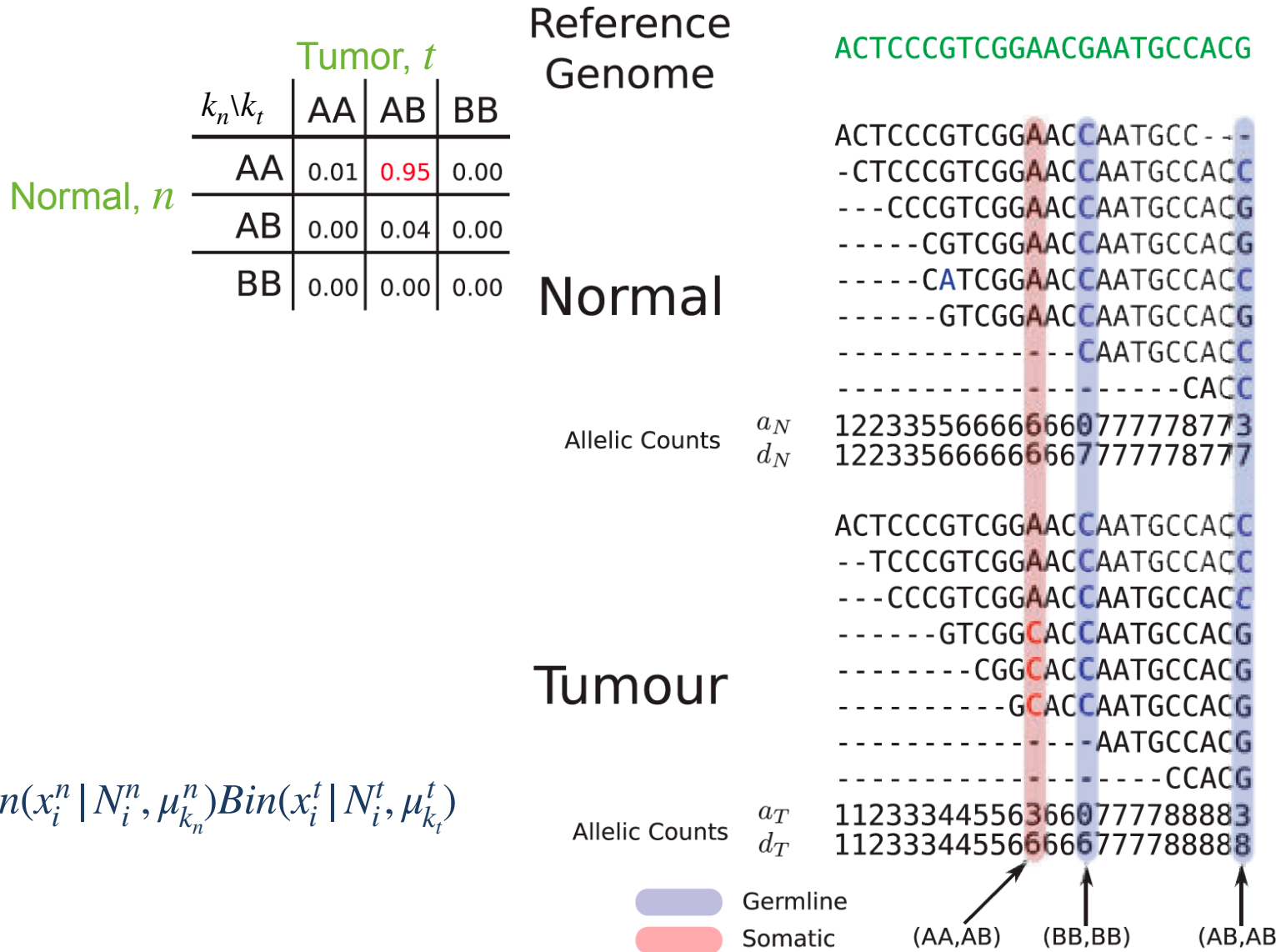
- 9 mixture weights  $\pi_{(k_n, k_t)}$

## 3. Joint binomial mixture model

- 9-component mixture model

$$p(x_i^n, x_i^t | N_i^n, N_i^t, \mu_{1:K}^n, \mu_{1:K}^t) = \sum_{k_n=1}^K \sum_{k_t=1}^K \pi_{(k_n, k_t)} \text{Bin}(x_i^n | N_i^n, \mu_{k_n}^n) \text{Bin}(x_i^t | N_i^t, \mu_{k_t}^t)$$

- with 9 parameter tuples  $(\mu^n, \mu^t)$



# Homework #5: Single-nucleotide Genotype Caller

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Implement a standard binomial mixture model described in Lecture 2.

- Learn the parameters and infer the genotypes
- Annotate the mutation status for a set of genomic loci.
- Expected outputs for each question will be provided so that you can check your code.
- RStudio Markdown and Python Jupyter Notebook templates provided.

**Due: May 8th**

Office Hours with Anna-Lisa Doebley ([adoebley@uw.edu](mailto:adoebley@uw.edu))

Zoom Meeting ID: 446 356 7725      Password: GS541

- Monday, May 4, 2-3pm
- Wednesday, May 6, 2-3pm